SAMBALPUR UNIVERSITY
STRUCTURE OF THE COURSE AND DETAIL SYLLABUS
FOR M.A./M.SC. MATHEMATICS PROGRAM, 2024-26
Passed in Board of studies in Mathematics meeting held on 09/08/2024

	SEMESTER-I		Mark distribution	Total
MAT- C-411	REAL ANALYSIS	4 CREDITS	80+20	100
MAT- C-412	COMPLEX ANALYSIS	4 CREDITS	80+20	100
MAT- C -413	ALGEBRA-1	4 CREDITS	80+20	100
MAT- C -414	TOPOLOGY	4 CREDITS	80+20	100
MAT- C -415	MATLAB	2 CREDITS	80+20	100
MAT- C -416	PROGRAMMING LABORATORY-I (MATLAB)	2 CREDITS	40+40+20 Record+practical+viva	100
	TOTAL	**20 CREDITS**		
	SEMESTER-II			
MAT- C -421	MEASURE THEORY AND INTEGRATION	4 CREDITS	80+20	100
MAT- C -422	ORDINARY DIFFERENTIAL EQUATION	4 CREDITS	80+20	100
MAT- C -423	ALGEBRA-II	4 CREDITS	80+20	100
MAT- C -424	DIFFERENTIAL GEOMETRY	4 CREDITS	80+20	100
MAT- C -425	PYTHON LANGUAGE	2 CREDITS	80+20	100
MAT- C -426	PROGRAMMING LABORATORY-II (PYTHON)	2 CREDITS	40+40+20 Record+practical+viva	
	TOTAL	**20 CREDITS**		
	SEMESTER-III			
MAT- C -511	FUNCTIONAL ANALYSIS	4 CREDITS	80+20	100
MAT- C -512	PARTIAL DIFFERENTIAL EQUATION	4 CREDITS	80+20	100
MAT- C -513	MATHEMATICAL METHODS	4 CREDITS	80+20	100
MAT- C -514	MATRIX ANALYSIS	2 CREDITS	80+20	100
MAT- C -515	PROGRAMMING LABORORATY-III (LATEX)	2 CREDITS	40+40+20 Record+practical+viva	100
MAT- E -516	ELECTIVE 1	4 CREDITS	80+20	100
	TOTAL	**20 CREDITS**		
	SEMESTER-IV			
MAT- C -521	OPTIMIMIZATION TECHNIQUE	4 CREDITS	80+20	100

MAT- C -522	PROBABILITY AND STOCHASTIC PROCESS	**4 CREDITS**	80+20	100
MAT- C -523	PROJECT/DISSERTATION (WITH VIVA VOCE)	4 CREDITS	20 (valuation in 3rd sem)+30 viva+ 50Dissertation work-	100
MAT- E -524	ELECTIVE-2	**4 CREDITS**	80+20	100
MAT- E- 525	ELECTIVE-3	4 CREDITS	80+20	100
	TOTAL	**20 CREDITS**		
ESDMS419	ENVIRONMENTAL STUDIES AND DISASTER MANAGEMENT (TO BE OFFERED IN 1ST SEMESTER)	2 CREDITS		
IDCMATH429	FOUNDATIONS IN MATHEMATICS (INTER DEPARTMENTAL COURSE, TO BE OFFERED IN 2ND SEMESTER)	3 CREDITS	80+20	100
EDPS 439	ENTREPRENEURSHIP DEVELOPMENT PROGRAM (TO BE OFFERED IN 3RD SEMESTER)	2 CREDITS		
MOOC MATH	A STUDENT WILL TAKE MOOC COURSE OF THREE CREDITS DURING SECOND OR THIRD SEMESTER FROM A LIST TO BE GIVEN BY THE DEPARTMENT /HOD DEPENDING ON THE AVAILABLE COURSES WITH OUT REPEATING ANY COURSE.THE STUDENT HAS TO SUBMIT THE DOCUMENT IN SUPPORT OF UNDERTAKING THE MOOC COURSES TO THE RESPECTIVE DEPARTMENT FOR FINAL RESULT.	3 CREDITS		
	GRAND TOTAL	**90 CREDITS**		
NON CREDIT COURSES	YUBA SANSKAR(As per Guideline)			
	NCC/NSS/SPORTS/PERFORMING ARTS/YOGA (ONE HAS TO BE ALLOTED TO THE STUDENT BY THE AUTHORITY DEPENDING ON NUMBER OF APPLICATION,MAXIMUM			

	CAPACITY AND PREFERENCE)			
	Alternative to MOOC COURSE NUMERICAL ANALYSIS	3 credits	80+20=100	

* **The Electives in different semesters are chosen from various GROUPS given in ScheduleA**.

LIST OF ELECTIVES

SCHEDULE = A

(Each Elective is of 4 Credits)

The Department will offer electives in Semester-III and Semester-IV from the following list. Students have to choose **one out of the electives offered by the Department in each Group**. Department may offer more than one electives in each group depending on the availability of expertise and faculty members.

GROUP I(Elective- I in Semester III will be one of following courses)

1. ADVANCED COMPLEX ANALYSIS
2. COMBINATORICS
3. FOURIER ANALYSIS
4. NUMBER THEORY AND FOUNDATIONS OF CRYPTOGRAPHY
5. MECHANICS
6. THEORY OF COMPUTATIONS

GROUP II (Eective -II of Semester IV will be one of following courses)

7. ALGEBRAIC TOPOLOGY
8. CRYPTOGRAPHY
9. DISCRETE DYNAMICAL SYSTEMS
10. GEOMETRIC FUNCTION THEORY
11. GRAPH THEORY
12. WAVELETS
13. NUMRICAL SOLUTION TO PARTIAL DIFFERENTIAL EQUATION

GROUP III (Eective- III of Semester IV will be one of following courses)

14. ANALYTICAL NUMBER THEORY
15. ADVANCED LINEAR ALGEBRA
16. DATA STRUCTURE
17. FRACTAL GEOMETRY
18. FLUID MECHANICS
19. MATHEMATICAL MODELLING
20. OPERATOR THEORY

Signature

Semester -I

REAL ANALYSIS

Course No MAT- C-411 **Credits -4**

Objective:

After a first course in real analysis in undergraduate program ,the ideas of uniform continuity, uniform convergence and approximation by polynomials are crucial in analysis. In addition to the Functions of bounded variation and their integrators, the student has to learn differentiating functions from R^n to R^m. The techniques of integration of a function with respect to another function and the basic ideas of finding a Fourier series are also included .

Expected Outcomes:

After studying the course

1. The student will understand and solve problems of uniform continuity, uniform convergence and will also test whether a function is of bounded variation or not.
2. The student will learn and solve problems about partial derivatives, directional derivatives ,Jacobians, Inverse and implicit theorems .
3. The student will be able to calculate Riemann Stieltjes Integrals.
4. The student will be able to find the Fourier series and apply it.

UNIT-I

Convergence of a sequence of real numbers, Continuous functions, Uniform continuity,Examples, pointwise and uniform convergence, tests of uniform convergence,Cauchy criterion, Weierstrass M Test, uniform convergence and continuity,uniform convergence and relation to integration and derivatives Weierstrass approximation theorem, power series,functions expressible as power series.

UNIT-II

Differentiation in R^n, Partial derivatives, Directional derivatives,sufficient condition for differentiability, chain rule, ,Mean value theorem, Jacobians, Contraction mapping principle, inverse function theorem, implicit function theorem, rank theorem, differentiation of integrals, Taylor theorem in many variables.

UNIT-III

Function of bounded variation, examples, total variation, Function of bounded variation expressed as difference of increasing functions, rectifiable paths, Riemann Stieltjes Integrals, properties and techniques, sufficient condition for existence of the integral, Necessary condition for existence of the integral, Mean value theorem for Riemann stieltjes integrals, Reduction to Riemann integrals.

UNIT-IV

Basic concepts of Fourier series, Fourier series of even and odd functions, half range series, Fourier series on other intervals, orthogonal systems of functions, Theorem on best approximation, properties of Fourier coefficients, Riesz Fisher theorem, Riemann Lebesgue lemma., Dirichlet integral, Integral representation for the partial sum of a Fourier series**.** Convergence of Fourier series

The course is covered by

1. W.Rudin-Principles of Mathematical analysis-Mc Graw Hill, 3rd Ed,
2. T Apostol-Mathematical Analysis- Pearson; 2nd edition, .

Books for Reference:

3. Terrence Tao- Analysis-I - Hindustan book agency.
4. Terrence Tao -Analysis- II- Hindustan book agency

5. S C Malik ,Savita Arora –Mathematical Analysis- New Age International -5th edition

COMPLEX ANALYSIS

Course No MAT- C-412 **Credits -4**

Objective: The course is a second course in complex analysis as every undergraduate student learns basic complex analysis as a core course. The objective is to make the student understand both the theory and problem components of analytic functions, conformal mappings , complex integration theory,product developments and normal families

Expected Outcomes:

After studying this course the student will be able to

CO1. understand analytic function as a mapping on the plane, Mobius transformation and conformal mappings.

CO2. prove Cauchy theorem on various domains and learn the use of Cauchy integral formula and other results.

CO3. Find singularities and Evaluate contour integral using method of residues.

CO4. Learn about product development,analytic continuation and normal families.

Unit-I

Review of analytic functions and basic properties, stereographic projections, mappings of elementary functions and cross ratio, Bilinear transformations and its properties, conformal mapping.

Unit-II

Complex integration and simple version of Cauchy's theorem: Curves, parameterization, line integrals, Cauchy theorems (rectangle, triangle, circular disck), Cauchy integral formula, Liouville's theorem, Morera's theorem, Cauchy inequality, fundamental theorem of algebra, uniqueness and identity theorems, maximum modulus theorems, Gauss- mean value theorem, Schwartz lemma. Poisson integral formula.

Unit-III

Calculus of residue: Laurent series, Classifications of singularities, evaluation of real integral, argument principle, Rouche's theorem, Hurwitz's theorem, open mapping theorem.

Unit-IV

Infinite product, Weierstrass product development, Mittag-Leffler's theorem, Analytic continuation, Schwarz reflection principle, Normal families.

Course is covered by:

1. S Ponnusamy and Herb Silverman: Complex variables with Applications: Birkhauser, (2006) (Indian Edition 2012)

(Chapter-2: 2.4; Chapter-3; Chapter-4, Chapter-5: 5.1, 5.2; Chapter-7; Chapter-8; Chapter-9; Chapter-10:10.2; Chapter-11: 11.2; Chapter-12; Chapter-13: 13.1)

Books for References:

1. L. V. Ahlfors - Complex Analysis, McGraw Hill, 3rd Ed.,2017.
2. R V Churchill , J W Brown and R F Verhey- Complex Variables and

Applications, McGraw Hill, 9th Ed., 2013.
3. J. B. Conway - Functions of one Complex Variable, Springer; 2nd ed. 1978, 7th printing 1995 edition.
4. E. M. Stein and R. Shakarchi, Complex Analysis: Princeton University Press, New Jersey, (2003)

ALGEBRA-I

Course No MAT- C-413 **Credits -4**

Objective: The objective of the course is to augment the core courses offered in under graduate level in group theory and linear algebra in a different perspective.

Expected Outcomes:

After studying this course the student will be able to

CO1. Solve problems of basic group theory,group actions,automorphisms and sylow theory.

CO2. Understand problems of product and semi direct product of groups and solvable groups

CO3. Find eigen value and eigen vectors and calculate various canonical forms.

CO4. Handle problems of unitary, self adjoint, normal operators and bilinear forms.

UNIT-I

Series of groups: Composition series and the Holder program, Transposition and the alternating group, Group actions: Group actions and permutations Representations, Cayley's Theorem, The Class equation, Automorphisms, The Sylow Theorems, The simplicity of alternating group.

UNIT-II

Direct and semi Direct products and abelian groups: Direct products, The Fundamental Theorem of finitely generated abelian groups, Recognizing Direct product, semidirect product, Further topics in Group Theory: p-groups, Nilpotent groups, Solvable groups, A word on free Groups

UNIT-III

Review of vector space fundamentals,matrix representation of linear transformations ,Eigenvalue and eigen vectors,Minimal polynomial, diagonalisation ,triangulable operators, nilpotent form, Jordan canonical form,rational canonical form,

UNIT-IV

Inner product spaces orthogonality . Adjoint of a linear transformation, unitary operators, Self adjoint and normal operators,bilinear forms,matrix of a bilinear form, classification of bilinear forms.

.

The course is covered by

1. David S. Dummit, Richard M. Foote, Abstract Algebra, 3rdd Paperback, Wiely, 2011.
2. V Sahay and V Bist : Linear Algebra ,Narosa publishing House , second edition

Books for reference

3. I.N. Herstein: Topics in Algebra , John Wiley and Sons; 2nd edition
4. J. B. Fraleigh: A first Course in Algebra, Pearson, 7th Ed., 2013.
5. J. Gallian: Contemporary Abstract algebra,Brooks/Cole ;8th edition
6. Hoffman and Kunz: Linear Algebra, Prentice Hall

7. Rao and Bhimasankaran: Linear algebra, Hindustan publishing house

TOPOLOGY

Course No MAT- C-414 **Credits -4**

Objective: This is an introductory course in Topology. The objective of this course is to have knowledge on topological spaces,Continuity, connectedness, compactness and separation axioms. Topology on Quotient spaces, Product spaces and metric spaces are also discussed.. The student will also learn on basic ideas of algebraic topology in homotopy, fundamental groups and covering spaces.However the thrust is on learning the point set topology.

Expected Outcomes:.

After taking the course the student will be able

CO1. To understand the concept of a topological space, basis, subbasis with various examples and to understand new topologies like product topology, quotient topology, metric topology etc .

CO2. To solve problems involving continuous maps , homeomorphisms between two spaces , connectedness and compactness.

CO3 To deal with Hausdorff, regular, normal ,separable, first and second countable spaces and Lindelöf spaces.

CO4 To understand homotopy, fundamental groups, and covering spaces.

UNIT I

Cartesian product of a family of sets, Axiom of choice and its equivalents (without proof) , Topological spaces, examples, open sets, closed sets , basis and subbasis for a topology, standard topology. Lower limit topology, order topology, Dictionary order, product topology on $X \times Y$,subspace topology, closed sets and limit points, interior of sets, Continuous functions, homeomorphisms,

.

UNIT-II

Box topology ,Product topology ,Metric topology, standard bounded metric,Euclidean metric square metric,uniform topology,sequence lemma,uniform limit theorem, Quotient topology, Connected spaces, Examples, Local connectedness, Path-connectedness, connected subsets of real line

Unit-III

compact Spaces, Examples and results, compact subsets of real line, locally compacts spaces, sequential compactness, limit point compactness, Countability axioms, First and second countable spaces, separable and Lindolf spaces, Separation axioms, Regular & completely regular space, normal spaces, Urysohn Lemma,

Unit-IV

Urysohn metrization theorem Tietze extension theorem,Tychonoff Theorem, , Homotopy, path homotopy,Fundamental Group, covering space, definition and examples, fundamental Group of circle.

,

The course is covered by

1. Munkres J R - Topology, A First Course: Pearson; 2nd edition, 2000.

Books for reference

2. K. D. Joshi : Introduction to General Topology (Wiley Eastern Limited).
3. M A Armstrong. Basic Topology. Springer, 1983.
4. O Viro, O Ivanov, V Kharlamov, and N Netsvetaev. Elementary Topology, a problem Text book,American Mathematical society.

.

MATLAB

Course No MAT- C-415 **Credits -2**

Objectives:

MATLAB has become essential in many undergraduate courses where practical component and computation is there. The objective of this couse is to train students in fundamentals of MATLAB tool.The student should understand MATLAB graphic feature and its application.This will promote new teaching model that will help to develop programming skill and techniques to solve problems.

Course Outcomes

After learning this course a student can

1.Use basic MATLAB tools

2.Plot different graphs in two-dimensions and three-dimensions.

3.Use the inbuilt array structures for calculations of algebra of matrices and solve the system of equations through various numerical methods.

4.Use different control flows for the writing of the simple programs and explore various applications to Numerial analysis and differential equations

Unit – I

Basics of MATLAB, Input – Output, File trypes – Platform dependence – General commands. Interactive Computation: Matrices and Vectors – Matrix and Array operations – Creating and Using Inline functions – Using Built-in Functions and On-line Help – Saving and loading data – Plotting simple graphs.

Unit – II

Programming in MATLAB: Scripts and Functions – Script files – Functions files- Language specific features – Advanced Data objects. Applications – Linear Algebra – Matrices: Eigenvalues and Eigenvectors, Similarity Transformation and Diagonalization, Curve fitting and Interpolation – Data analysis and Statistics

Unit – III

Graphics: Basic 2-D Plots – Using subplot to Layout multiple graphs - 3 - D Plots – Handle Graphics – Saving and printing Graphs – Errors Application– Symbolic math:creating symbolic objects and expression, changing the form of an existing symbolic expression:collect,expand,factor,simplify,pretty command

Unit –IV

. Applications to Numerical differentiation and integrations, roots of polynomials, finding maximum and minimum of a function, Ordinary differential equations,Analytic solution of ODE,boundary value problem,PDE:-pdepe command

Introduction to working with modules in MATLAB,

The course is covered by

1. Rudra Pratap, Getting Started with MATLAB – A Quick Introduction for Scientists

and Engineers, Oxford University Press, 2003.
Books for Reference
2. Amos Gilot:MATLAB ,An Introduction with applications, Wiley ,4th Edition, 2011
3. Yang, W. Y., Cao, W., Chung, T. and Morris, J. Applied Numerical Methods using MATLAB. John Wiley Interscience, 2005

PROGRAMMING LAB-I(MATLAB)

Course No MAT- C-416 **Credits -4**

Objective:
These practicals add to their undergraduate training of writing MATLAB CODES for various Mathematical problems

Expected Outcome:
1. To learn to write codes using basics of MATLAB
2. To write code for problems from calculus and series sums.
3. To Write MATLAB codes for problems linear Algebra
4. To write MATLAB code for finding roots of equations, for problems in Numerical analysis .

List

GROUP-A

1.Write MATLAB code to find a root of the equation $x^3-5x+1 = 0$ by using Bisection method.

2. Write MATLAB code to find the solution of a nonlinear equation $\tan(\pi-x)-x = 0$ by using Bisection method.

3. Using Bisection method find the roots of the following equation taking up to 50 iteration. $X^2 + 2x-2 = 0$.

4. Write MATLAB code to find a root of the equation $x^3 -5x + 1 = 0$ by using Secant method.

5. Write MATLAB code to find a root of equation $\sin x = e^x - 5$ by using Newton-Raphson method.

6. Write MATLAB code to find a root of the equation $\cos x-xe^x = 0$ by using Newton Raphson method.

7. Write MATLAB code to find a root of the equation $x^3 - 5x + 1 = 0$ by using Regula-Falsi Method.

8. Write MATLAB code to find a root of the equation $\cos x - xe^x = 0$ by using Regula-Falsi Method.

9. Write MATLAB code to find the approximate value of $\int_0^{\frac{\pi}{2}} \boldsymbol{sinx dx}$ by using the trapezoidal rule. Also compute the error.

10. Write MATLAB code to find the approximate value of $\int_0^1 \boldsymbol{e^{-x} dx}$ by using the trapezoidal rule. Also compute the error.

11. Write MATLAB code to find the approximate value of $\int_0^1 \frac{1}{1+x} \boldsymbol{dx}$ by using Simpson's 1/3rd Rule.

12. Write MATLAB code to find the approximate value of $\int_0^1 \frac{1}{1+x} \boldsymbol{dx}$by using Simpson's 3/8 Rule.

13. Write MATLAB code to find the solution of the system of equations : $4x_1 +x_2 +x_3 = 2$; $X_1 + 5x_2+2x_3 = -6$; $x_1+2x_2+3x_3 = -4$ by using Gauss-Jacobi iteration method with $[0.5,-0.5,-0.5]^T$.

14. Write MATLAB code to find the solution of the system of equations $4x_1 + x_2 + x_3 = 2$; $X_1 + 5x_2 + 2x_3 = -6$; $x_1 + 2x_2 + 3x_3 = -4$ by using Gauss-Seidal iteration method.

15. Write MATLAB code to find the solution of the system of equations $x_1 - 2x_2 + x_3 = 0$; $2x_1 + x_2 - 3x_3 = 5$; $4x_1 - 7x_2 + x_3 = -1$ by using Gauss elimination method.

16. Write MATLAB code to find the value of a function f at 2.2 using Lagrange interpolation method, where f(0) = 1, f(1) = 3 and f(3) = 55.
17. Write MATLAB code to find the value of a function f at 13 using Lagrange interpolation method, where f(5) = 12, f(6) = 13, f(9) = 14 and f(11) = 16.

GROUP B

1. Write MATLAB code to plot y = sin(1/x) and y = sinx + cosx on a single figure window.
2. Write a MATLAB program to plot y = cos(1/x) and y = sin2x−cos(x/2) on a single figure window.
3. Write a MATLAB program to plot y = sin(1/x) and y = x^2 + expx on a single figure window.

4. Write MATLAB code to Sketch the parametric curve Trochoid, cycloid and epicycloid.
5. Write MATLAB code to Sketch Ellipsoid, Hyperboloid and sphere of radius 2.
6. Write MATLAB code to the parametric curve Trochoid, cycloid and epicycloid.
7. Write MATLAB code to Sketch Ellipsoid, Hyperboloid and sphere of radius 5.3.
8. Write a MATLAB code to plot surface of revolution around x-axis.
9. Write a MATLAB code to plot sin(4x−1), 1/(ax+b)and |−7x + 78|.
10. Write a MATLAB code to plot sin(ax + b), 1/(ax+b) and |ax + b|.
11. Write a MATLAB code to plot the surface z= $\frac{xy^2}{x^2+y^2}$ for $-1 \le x \le 3$ and $1 \le y \le 4$.
12. Write MATLAB code to plot a function f(x,y) = $\frac{xy}{x^2+y^2}$.
13. Write MATLAB code to plot the surface z = $\frac{xy(x^2-y)}{x^2+y^2}$ for $-3 \le x \le 3$ and $-3 \le y \le 3$.
14. Write MATLAB code to plot the surface z = $\frac{y^2(x^2-y^2)}{x^2+y^2}$ for $-2 \le x \le 2$ and $-2 \le y \le 2$.
15. Writa a MATLAB code to trace Parabola and Ellipse.
16. Write a program to find one-norm and two-norm of a given 3×3 Martix.
17. Write a program to find one-norm and two-norm of a given 5×5 Martix.
18. Write MATLAB code to plot the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ where a = b = 5 and c = 3.

.

Students are required to maintain practical records and get practical questions from each group in exam.

Semester - II

MEASURE THEORY AND INTEGRATION

Course No MAT- C-421 **Credits -4**

Objectives: Measure Theory formalises and generalises the notion of integration. It is fundamental to many areas of mathematics and probability and has applications in other fields such as physics and economics. Students will be introduced to Lebesgue measure and integration, signed measures, the Hahn-Jordan decomposition, the Radon-Nikodym derivative, conditional expectation, Borel sets and standard Borel spaces, product measures, and the Riesz representation theorem.

Expected Outcomes: After completing this subject, students will understand the fundamentals of measure theory and be acquainted with the proofs of the fundamental theorems underlying the theory of integration. They will also have an understanding of how these underpin the use of mathematical concepts such as volume, area, and integration and they will develop a perspective on the broader impact of measure theory in Ergodic theory and have the ability to pursue further studies in this and related areas.

Unit-I

Lebesgue Outer measure, measurable sets, regularity, measurable functions, Borel and Lebesgue measurability, non-measurable sets, integration of nonnegative functions, simple functions, Lebesgue integration of simple, Approximation of measurable functions by simple functions, Fatou's lemma, monotone convergence theorem

G. de Barra: Measure Theory and Integration, Chapter 2(2.1to2.5), Chapter 3(3.1)

Unit-II

General integrals, properties, Lebesgue dominated convergence theorem, integration of series, Riemann and Lebesgue integrals, differentiation, Dini derivatives, Lebesgue differentiation theorem, Differentiation and integration, Abstract measure spaces, measure and outer measure.

G. de Barra: Measure Theory and Integration, Chapter 3(3.2 to 3.4), Chapter 4(4.1, 4.4, 4.5), Chapter 5(5.1)

Unit-III

Extension of a measure, uniqueness of the extension, completion of a measure, Integration with respect to a measure. Modes of convergence, convergence in Measure, almost uniform convergence, fundamental in measure convergence, Egorov's theorem, diagrams and counter examples.

G. de Barra: Measure Theory and Integration, Chapter 5(5.2, 5.3), Chapter 7(7.1, 7.2, 7.3, 7.4)

Unit-IV

Signed measure, absolute continuity, Hahn decompositions, Jordan decomposition, Lebesgue decomposition, Radon-Nikodym theorem, Applications of Radon Nikodym Theorem Product measure, Fubini Theorem.

G. de Barra: Measure Theory and Integration, Chapter 8(8.1, 8.2, 8.3, 8.4), Chapter 10(10.1, 10.2)

The course is covered by

1. De Barra. G.: Measure Theory and Integration (New age International), 1981

Books for Reference

1. Royden, H. L: Real Analysis-Pearson, 4th Ed., 2010.
2. Aliprantis C D ,Burkinshaw O,Principles of Real Analysis, Elsevier 2011
3. Rudin W: Real and Complex Analysis.(Tata McGraw Hill of India), 3rd Ed, 1986
4. Hewitt and Stromberg: Real and absratct analysis-Springer, 1975.

ORDINARY DIFFERNTIAL EQUATIONS
Course No MAT- C-422 **Credits -4**

Objective: Differential Equations introduced by Leibnitz in 1676 models almost all physical, biological, Chemical, Socio-economic system in nature. The objective of this course is to familiarize the students with qualitative behaviour of solutions differential equations along with existence and uniqueness problems. Also, students will get an insight to the concept of stability on differential equations of first and second order.

Expected Outcomes: A student completing the course will be able to understand the various type of behavior of solutions of differential equations and will be able to model problems in nature using ODE. This is also prerequisite for taking other core courses in partial differential equations, Stability theory, Oscillation theory, Evolution equations, Dyncmical systems, Bifurcation theory, Mathematical modeling etc.

Unit-I

Review of second order differential equations: methods, homogeneous, nonhomogeneous.

Oscillation of Second Order Linear Differential Equations:- Fundamental results, Sturm's Comparison Theorem and Hille-Wintner type oscillation.

Second Order Boundary Value Problem:- Sturm-Liouville differential equation, eigen value problem, Green's function and Picard's Theorem.

Unit-II

Existence and Uniqueness of Solutions:- Successive approximations, Picard's Theorem, Nonuniqueness of solutions, Continuation and dependence on initial conditions, Existence of solutions in the large.

Unit-III

System of Linear Differential Equations:- System of first order equations, Existence and uniqueness theorems, Fundamental matrix, Homogeneous and nonhomogeneous linear systems with constant coefficient.

Unit-IV

Stability:- Autonomous Systems, Stability for linear systems with constant coefficients, Stability for linear systems with variable coefficients, Linear plane autonomous systems, Perturbed systems, Method of Lyapunov for nonlinear systems, Limit cycles.

The course is covered by

1. S. G. Deo, V. Raghavendra: Ordinary Differential Equations
Tata McGraw-Hill Publishing Co. Ltd

Books for References

.

2. Richard Bellman: Stability Theory of Differential Equations
McGraw-Hill Book Company, Inc., New York
3. Tyn-Myint-U: Ordinary Differential Equations
Elsevier North-Holland.
4. G. F. Simmons: Differential Equations with Applications
McGraw-Hill International Editions
5. G. Birkhoff, G. C. Rota: Ordinary Differential Equations
John Wiley & Sons, New York

ALGEBRA-II

Course No MAT- C-423 **Credits -4**

Objective: As a second course in algebra the objective of this course is to have more knowledge on ring theory and to know about field theory. The concept of Galois theory in fields is central to theory of equations and is a must for all mathematics students. Understanding of these basic theories pave the way for any advance course in algebra.

Expected Outcomes: The knowledge on this course will provide the basis for further studies in advanced algebra like commutative algebra, linear groups, etc., which forms the basics of higher mathematics.

Unit-I

Review of Ring theory. Euclidean ring, Gaussian integers, Polynomial ring, Polynomial ring over rational field, Polynomial ring over commutative ring, Principal ideal domain, Unique factorization domain

Unit-II

Field Theory: Basic Theory of Field Extensions, Algebraic Extensions, Classical Straight edge and Compass constructions, Splitting Fields and Algebraic closures

Unit-III

Separable and inseparable extensions, Cyclotomic polynomials and extensions, Galois Theory: Basic Definitions, The fundamental theorem of Galois theory

Unit-IV

Finite Fields, composite and simple extension, normal extension, Galois groups of polynomials, solvability by radicals.

The course is covered by

1. David S. Dummit, Richard M. Foote, Abstract Algebra, 3rd ed, Wiely, 2011.

Books for reference:

2. I. N. Herstein, Topics in Algebra, John Wiley and Sons, 2nd Revised edition, 1975.
3. J. B. Fraleigh, A first Course in Algebra, Pearson, 7th Ed., 2014.
4. J. Gallian, Contemporary Abstract algebra, Brooks/Cole Pub Co; 8 edition (2012).

DIFFERENTIAL GEOMETRY

Course No MAT- C-424 **Credits -4**

Objective: After a course in Analytic Geometry and Differential geometry of curves at undergraduate level, Differential Geometry is a core component of a post graduate syllabus which introduces the methods of differential manifolds, tensor analysis, vector fields, Lie Group, Lie Algebra etc. The objective is to prepare the students for further coursework and research in geometry in future.

Expected Outcome: After completing this course, a student can opt for a course on Lie Group, Lie Algebra, Symplectic Geometry, Poisson Geometry, Global Analysis, Several Complex Variable, Hyperbolic Geometry, Projective and Algebraic Geometry and all these courses are main component for Mathematical Physics, Relativity, Cosmology and Standard Models.

Unit-I

Review of calculus in R^n, locally Euclidean spaces,charts,atlases.manifolds, Differential manifolds, Examples, differentiable function on a manifold, Tangent vector on a manifold an equivalence class of curves and as a directional derivative operator, algebraic approach, differential of smooth maps, immersion, submersion, Submanifolds, tangent space at a point of the manifold, cotangent spaces,Bases for tangent space and cotangent space

Unit-II

vector fields on differentiable manifolds, Integral curve, Lie bracket of vector fields, Definition and examples of Lie algebra, Definition and examples of Lie groups,
Lie algebra of a Lie group

Unit-III

Multi linear Algebra: Dual space,tensor products, tensor of type(r.s), Operations with tensors, contractions, quotient law of tensors, metric tensor, associated tensors, symmetric and antisymmetric tensors, Exterior forms, differential forms Wedge product , Exterior Algebra, Exterior derivative, Exact forms, Closed forms. Integration on manifolds, Stoke's theorem

Unit-IV

Affine connection of manifolds, parallel transport, curvature and torsion tensor of an affine connection, Intrinsic derivative, covariant derivative, Riemannian metric, Riemannian manifold, Fundamental theorem of Riemannian Geometry, Levi Civita Connection, Riemann Curvature tensor and properties, Bianchi identities, Ricci tensor, Scalar curvature

The course is covered by

1. U.C De & A.A Shaikh, Differential Geometry of Manifolds, Narosa, 2009
2. Wilmore- Differential and Riemannian geometry, Oxford University Press, 1996
3. Tu L W, Introduction to manifolds,Springer International edition

Books for Reference:

.

1. Chern,Chern,and Lam,Lectures in Differential Geometry,Wolrd Scientific(Indian Edition)
2. Warner-Foundations of differential geometry and Lie groups Springer, 1983.
3. Nicolaescu Liviu I, Geometry of Manifolds World scientific. Indian Edition 2021
4. Boothby - An introduction to differential and Riemannian geometry, Academic Press; 2 edition , 2002

PYTHON LANGUAGE

Course No MAT- C-425 **Credits -2**

PYTHON LANGUAGE

UNIT-I

INTRODUCTION DATA, EXPRESSIONS, STATEMENTS

Introduction to Python and installation, variables, expressions, statements, Numeric data types: Int, float, Boolean, string. Basic data types: list operations, list slices, list methods, list loop, mutability, aliasing, cloning lists, list parameters. Tuple --- tuple assignment, tuple as return value, tuple methods. Sets: operations and methods. Dictionaries: operations and methods.

UNIT II

CONTROL FLOW, LOOPS, FUNCTIONS Conditionals: Boolean values and operators, conditional (if), alternative (if-else), chained conditional (if elseif-else); Iteration: statements break, continue. Functions: function and its use, pass keyword, flow of execution, parameters and arguments. . Python arrays: create an array, Access the Elements of an Array, array methods.

UNIT III

FILES,EXCEPTIONS File I/O, Exception Handling, introduction to basic standard libraries, Installation of pip, Demonstrate Modules: Turtle, pandas, numpy, pdb, Explore packages. Object, Class, Method, Inheritance, Polymorphism, Data Abstraction, Encapsulation.

UNIT IV

Linear Algebra, Linear Equations, Eigen Values and Eigen Vectors, Taylor Series, Fourier Transform.

TEXT BOOK

1. R. Nageswara Rao, Core Python Programming‖, dreamtech
2. Qingkai Kong Timmy Siauw Alexandre M. Bayen , Python Programming and Numerical Methods A Guide for Engineers and Scientists.

PROGRAMMING LABORATORY II (PYTHON LANGUAGE)

Course No MAT- C-426 **Credits -2**

Objective:

These practicals augment to the theory course taught in PYTHON programming to enhance the numerical skills.

Expected Outcome:

1. To learn to write codes using basics of Python programming
2. To write code for problems from calculus ,linear Algebra and Numerical analysis .

NOTE:Though any distribution of Python 3 software can be used for practical sessions, to avoid difficulty in getting and installing required modules like numpy, scipy etc, and for uniformity, the Python 3 package Anaconda 2018.x (https://www.anaconda.com/distribution/#downloadsection) may be installed and used for the practical sessions.

1. Finding the limit of functions,
2. Finding the derivative of functions, higher-order derivatives
3. Finding the maxima and minima
4. Finding the integrals of functions.
5. Verify the continuity of a function at a point
6. Find Area between two curves

7. Finding the length of a curve.
8. Polynomial Interpolation by Lagrange's Method, Newton's Method
9. Find Roots of Equations by Method of Bisection and Newton-Raphson Method
10. Gauss Elimination Method (excluding Multiple Sets of Equations),
11. Doolittle's Decomposition Method only from LU Decomposition Methods
12. Numerical Integration
13. By Newton-Cotes Formulas
14. Trapezoidal rule,
15. Simpson's rule and Simpson's 3/8 rule
16. Finding eigen value and eigen vectors
17. Finding Talor series
18. Finding Fourier transforms
19. Solving linear equations
20. Attempting problems in unit III and IV in Python Theory course

A practical record book should be maintained by the student

Semester - III

FUNCTIONAL ANALYSIS
Course No: MAT- C -511 Credits-4

Objective:The aim of the course is to introduce students to L^p spaces,Banach Spaces, Hilbert Spaces etc which has got numbers of appications in Mathematical Physis,Advanced Analysis etc. Students will also be exposed to Bounded Linear Operators and unbounded operators with basics of spectral Analysis.
Prerequisite: Real Analysis, Metric spaces and Measure Theory
Expected Outcome: On successful completion of the course, students can opt for courses like, Harmonic Analysis, Operator Theory Spectral Theory,quantum mechanics Scattering Theory, non-commutative geometry and operator algebra etc.

Unit - I
Review of Metric spaces, l^p and L^p spaces,Jensen, Holder and Minkowskii inequalities, Completeness of L^p, Denseness of $C_c(X)$ inL^p space, Normed spaces,Banach Spaces, Examples, Hamel basis,Schauder basis,Finite dimensional normed spaces, compactness and finite dimension.
The course is covered by

1. Chapter 3 Walter Rudin, Real and complex analysis Tata Mcgraw Hill

2. Chapter 2(2.2to 2.5) Kreyszig-Functional Analysis -John Wiley, 2013

Unit -II
inner product spaces, Hilbert spaces, examples, continuity of inner product, Schwarz inequality, orthogonal complements, direct sums, Projection theorem, orthnormal sets and sequences, Bessel inequality,Gram-Schimdt orthonormalizations, Fourier series expansion, Riesz-Fischer Theoremi,Parseval’s formula,Total orthonormal sets

,separable Hilbert spaces,,representation of functionals on Hilbert spaces,
The course is covered by

1. Chapter 3(3.1 to 3.6, 3.8) Kreyszig-Functional Analysis -John Wiley, 2013

Unit – III

Linear operators. Range and null space, bounded and continuous linear operators,examples,linear functionals, linear operators and functional on finite dimensional spaces. normed space of operators, dual space examples,Hahn Banach Theorem,Appplications,,adjoint operator ,reflexive spaces Baire's category theorem, Uniform boundedness principle, strong and weak convergence, Open mapping Theorem, Closed graph theorem,

. The course of this unit is covered by

1. Chapter 2(2.6-2.10 and chapter4(4.1to4.8 4.12,4.13) Kreyszig-Functional Analysis -John Wiley, 2013

Unit - IV

Spectral theory in finite dimensional normed spces, basic concepts of spectral theory,spectral properties of bounded linear oprators,spectral theorem for a polynomial, Banach algebra, definition, Examples, Compact operators on normed spaces, Unbounded linear Operator.

The course is covered by

1. Chapter5.1, 7(7.1to7.7)8.110.1 Kreyszig-Functional Analysis -John Wiley, 1978

Books for reference

1. Limaye -Functional Analysis, 3rd Ed, 2014.
2. Goffman and Pedrick A first Course in Functional Analysis- Prentice Hall (1 June 1965)
3. Jain and Ahuja,Functional Analysis New Age International 2017
4. Ponnusamy S. ,Foundations of functional Analysis Narosa 2011

PARTIAL DIFFERENTIAL EQUATIONS
Course No: MAT- C -512 Credits-4

Objective: The objective of this course is to understand basic methods for solving Partial Differential Equations In the process students will be exposed to canonical forms of hyperbolic,elliptic and parabolic pdes and shall solve wave equation, heat equation, Laplace Equation for many cases. They will be exposed to Greens functions, Fourier transform methods and teory of finite differences They will also solve various boundary value problems.

Expected Outcomes: After completing this course, a student will be able to take advanced courses on wave equation, heat equation, diffusion equation, gas dynamics, non linear evolution equations and integrable models etc. All these courses are important in engineering and has industrial and defence application.

Unit - I

Review of First order pdes, classification and Cauchy problems, Linear Second order partial Differential Equations, Classification of Second order Partial Differential Equations., canonical forms of hyperbolic, parabolic and elliptic partial differential equations Origin of second order PDEs, vibrating string, vibrating membrane, waves in elastic medium, conduction of heat in solids, Laplace Equation(Cartesian, polar), diffusion equation , Solution of these by separation of variables.

Unit - II

One dimensional Wave equation, Vibration of an infinite string D'Alembert's solution, Vibrations of a semi finite string, Vibrations of a string of finite length, existence and uniqueness of solution, Boundary and initial value problems for two dimensional wave equation Heat (diffusion) equation, Heat conduction problem for an infinite rod, Heat conduction in a finite rod, existence and uniqueness of the solution, Solution using Fourier transform.

Unit - III

Boundary value problems for Laplace equation, Maximum and minimum principles, Uniqueness and continuity theorems, Dirichlet interior and exterior problem for a circle, Neumann interior problem for a circle, Dirichilet problem for a rectangle, Neumann problem for a rectangle, Kelvin's inversion theorem, Equipotential surfaces, Riemann method, Theory of Green's function for Laplace equation, Dirichlet problem for Laplace equation using Green's function

Unit-IV

Solution of heat equation, wave equation by Green's function method, introduction to finite difference schemes, Convergence and consistency, Stability, The Courant-Friedrichs- Lewy Condition, Von Neumann Analysis

The course is covered by

1. Tyn-Myint-U - Partial Differential Equations of mathematical Physics, Elsevier North Holland New York, 1978.
2. J. C. Strikwerda, Finite Difference Schemes and Partial Differential Equations, Second Edition, SIAM , 2004. (UNIT IV)

Books for reference

3. K Shankar Rao, Introduction to partial differential equations,PHI learning private Ltd 2011
4. J. N. Sharma, K. Singh, Partial Differential Equations for Engineers and Scienists, Narosa, 2^{nd} Edition.

MATHEMATICAL METHODS
Course No: MAT- C -513 Credits-4

Objective: The objective of this course is to prepare a student in basics of Integral transforms, Integral equations and calculus of variations. These tools have engineering applications. Fourier transform and Laplace transform help in studying differential equations and other engineering problems. Calculus of variations and Euler equations are essential in understanding many physical problems and optimization problems.

Expected outcomes: A student trained in this course can opt for courses like digital signal processing, variational analysis, Wavelets. This exposes the application of mathematics to various real life problems.

Unit-I

Laplace transforms: Definitions, Properties, Laplace transforms of some elementary functions, Convolution Theorem, Inverse Laplace transformation, Applications. Fourier transforms, Definitions, Properties, Fourier Transforms of some elementary functions, Convolution, Fourier transforms as a limit of Fourier Series, Applications to PDE.

Unit-II

Volterra Integral Equations: Basic concepts, Relationship between Linear differential equations and Volterra integral equations, Resolvent Kernel of Volterra Integral equations, Solution of Integral equations by Resolvent Kernel, The Method of successive approximations, Convolution type equations, Solutions of integral differential equations with the aid of Laplace transformations.

Unit-III

Fredholm Integral equations: Fredholm equations of the second kind Fundamental, Iterated Kernel, Constructing the resolvent Kernel with the aid of iterated Kernels, Integral equations with degenerate Kernels, Characteristic numbers and eigen functions, solution of homogeneous integral equations with degenerate Kernel- non homogeneous symmetric equations Fredholm alternative.

Unit-IV

CALCULUS OF VARIATIONS: Extremal of Functionals : The variation of a functional and its properties , Euler's equations, Field of extremals, Sufficient conditions for the Extremum of a Functional conditional Extremum Moving boundary problem, Discontinuous problems, one sided variations, Ritz method.

The course is covered by

1. Lokenath Debnath., Integral Transforms and their applications CRC press.(unit-I)
2. A. J. Jerri, Introduction to Integral Equations with Applications, John-Wiley & SONS, INC., 1999(Unit-II, Unit-III)
3. AS Gupta Calculus of Variations with applications Prentice Hall of India (Unit IV)

Books for Reference:

1. Sneddon I., The use of Integral Transformations (Tata McGraw Hill), 1972.
2. Murray R Spiegel, Schaum's Series, Laplace Transforms, 1965.
3. Gelfand and Fomin, Calculus of Variations, Dover Pub, 2003.
4. Krasnov, Problems and Exercises in Calculus of Variations(Mir Publ), 1970
5. Ram P Kanwa, Linear Integral Equations (Academic Press), 2013.
6. A. J. Jerri, Introduction to Integral Equations with Applications, John-Wiley & SONS, INC., 1999.

MATRIX ANALYSIS

Course No: MAT- C -514 Credits-4

Unit I
Matrix polynomials: commuting matrices,matrix decompositions,Annihilating polynomials of matrices,Special types of matrices: Idempotennce,nilpotence,involution and projection, tridiagonal matrices ,

Unit-II ,
circulant matrices,vander mode matrices ,Hadamard matrices,permutation and doubly stochastic matrices

Unit-III .
positive semi definite matrices, A pair of positive semi definite matrices, partitioned positive semi definite matrice , Schur compements and determintal inequalities Kronecker product and Hadamard product .

Unit-IV
Hermitian matrices,product of Hermitian matrices,the min max theorem and interlacing theorem,eigen value and singular value inequalities . •

The course is covered by 1, Fuzhen Fang, Matrix Theory , Springer international edition

Books for reference

1. Roger A. Horn and Charles R. Johnson, “Topics in Matrix Analysis”, Cambridge University Press, 1999

PROGRAMMING LABORATORY - III

Course No: MAT- C -515 Credits-4

LaTeX Programming

Objectives: To introduce students with a software that is being widely used for typesetting especially in Mathematics field. To make students know importance of this software for publishing research articles, papers, project reports and books and thereby help them to be comfortable with the software.

Course outcome- After taking this lab course a student will be able to prepare a mathematics thesis,lab document , research paper or report on his own.

Unit I: Installation of LaTeX

i) Installation of Kile and MikeTeX.

ii) Class and packages

iii) Latex programming and commands, sample packages

iv) Error messages : Some sample errors, list of LaTeX error messages

Unit II: Formatting of output document:

i) Fonts, symbols, indenting, paragraphs, line spacing, word spacing, titles and subtitles

ii) Document class, page style, parts of the documents, table of contents

iii) Command names and arguments, environments, declarations

iv) Theorem like declarations, comments within text,

Unit –III: Mathematical formulae:

i) Mathematical environments, math mode ,mathematical symbols

ii) Graphic package, multivalued functions, drawing matrices

iii) Tables, tables with captions

iv) References to figures and tables in text

Unit –IV: Drawing with LaTeX

i) picture environments

ii)extended pictures, other drawing packages

iii) Preparing book, project report in LaTeX.

Reference Book:

1. *Helmut Kopka, Patrick W.Daly, Guide to LATEX,* fourth edition, Addison Wesley Longman Limited 2004
2. D.F.Griffiths, D.J.Higham, Siam, Philadelpha, *Learning Latex* , 1997
3. Martin J. Erickson and Donald Bindner, *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*, CRC Press, Boca Raton, FL, 2011.
4. L. Lamport. *LATEX: A Document Preparation System, User's Guide and Reference Manual*. Addison-Wesley, New York, second edition, 1994.

Semester – IV

OPTIMIZATION TECHNIQUE
Course No: MAT- C -521 Credits-4

Objective:- The objective of this course to acquire knowledge in various optimization techniques other than the course taught in the undergraduate level. With this paper, students will be exposed to nonlinear programing problems and different methods, sensitive analysis and parametric programing problems. Also, students will be able to learn about queuing theory and queuing models useful in daily life.

Expected Outcomes:- After completing this course, a student will be comfortable to opt for M.Tech in computer science with the undergraduate Operation Research course in the mind. This course will help a lot to the students availing DRDO program. It is good to see the mathematics students coming forward for research in Operation Research which is also a research topic in Statistics.

UNIT-I: Review of LPP methods. Introduction to Sensitivity analysis, variation in cost and requirement vectors, coefficient matrix and applications. Parametric linear programming problem and examples.

UNIT-II: Introduction to Nonlinear Programming Problem (NLPP) and formulation, general NLPP, constrained optimization with equality and inequality constraints, saddle point, necessary and sufficient conditions for saddle point with NLPP.

UNIT-III: NLPP methods. Graphical method, Kuhn-Tucker conditions, quadratic programming, Wolfe's method, Beale's method, separable convex programming and algorithm.

UNIT-IV: Introduction to queuing system, operating characteristics, probability distribution, classification of queuing models, Poisson and Non-Poisson queuing systems.

The course is covered by

1. H. A. Taha; Operations Research, McMillan (5th edition), 1992.

Books recommended

2. G. Hadley; Linear Programming, Narosa, India.
3. N. S. Kambo; Mathematical Programming Techniques, Affilated to East-West Press, Pvt. Ltd.
4. F. S. Hiller, G. J. Lieberman, B. Nag, P. Basu; Introduction to Operation Research, Mc Graw Hill Education Pvt. Ltd., India.

PROBABILITY and STOCHASTIC PROCESS

Course No: MAT- C -521 Credits-4

Objective:

- To introduce probability concepts using measure theory.
- To illustrate basics of various random processes such as Markov Chains, Poisson processes, renewal processes and Brownian motion etc for application
Expected Outcomes:
- The training will train the students in various applications ofstochastics process in Mathematical finance, physical sciences
,communication engineering and computer science.

Unit-I

Sets and events,, algebra of ses,σ fields, Monotone class ,Borel fields. random variables measurable function Borel function and induced sigma field,limits of random variable. Probability measure and Probability space, properties of profanity measure, , induced probability space, probability distribution, Discrete and continuous random variables, Examples conditional probability measure, distribution function of random variable. Independence of events., expectation of random variables properties of expectation, moment generating function, , Markov inequality, Chebyshev inequality, Jensen, Holder and Murkowski inequalities Borel Cantelli Lemma

Unit-II

Convergence of a sequence of random variables, Types of convergence, Convergence in probability, Almost sure convergence, convergence in distribution, convergence in mean, monotone convergence theorem, dominated convergence theorem, Independence of events, independence of classes independence of random variables, Borel 0-1 law, Kolmogorov 0-1 Law, weak law of large numbers, strong law of large numbers Central limit theorems(CLT) CLT for iid random variables, Liapunov and Lindeberg,s theorem, Conditional probability, review of Radon Nykodym theorem, Conditional expectation, properties , Characteristic function and its properties,

Unit-III

Definition, examples and classification of random(stochastic) processes according to state space and parameter space. Stationary Processes: Weakly stationary and strongly stationary processes, Discrete-time Markov Chains, Transition probability matrix, Chapman-Kolmogorov equations;classification of states,Recurrence. Transience, Markov chain, , ergodicity, stationary distribution,random walk and gambler's ruin problem, Branching Processes

Unit-IV

exponential processes, Counting processes,Poisson processes, Continuous-time Markov Chains, birth-death processes, limiting probabilities Renewal Processes: Renewal function and its properties, renewal theorems,Brownian Motion,Wiener process as a limit of random

walk; definition and examples of martingales,

,

The course is covered by

1. BR Bhatt Modern Probability Theory, New age international publisher
2. Ross S.M.: Introduction to probability models, 11th edition,Elsevier, 2014.

Books for reference

1. Papoulis A.: Probability,Random variables and stochastic processes ThirdEdition Mc Graw Hill
2. Billingsley P: Probability and measure Third edition Wiley India 2008
3. Ross S M: A First Course in probability 4th edition,Prentice Hall,
4. Chow Y. and Teicher H: Probability Theory Springer Intrnational Edition
5. S Karlin and H M Taylor:A First Course in Stochastic Processes, Academic press
6. J. Medhi:Stochastic Processes, 3rd Edition, New Age International, 2009.

SYLLABUS OF ELECTIVES
ANALYTIC NUMBER THEORY

Objective

Analytic number theory aims to study number theory by using analytic tools such as Inequalities, Limits, Calculus, etc. The aims of this syllabus are to illustrate how general methods of analysis can be used to obtain results about integers and prime numbers, to investigate the distribution of prime numbers etc. After completion of this course the students are able to know distribution of primes using analysis, the basic theory of Riemann zeta function and related L-function, to understand the proof of Dirichlet's theorem, To know the basic properties of Bernoulli numbers and Bernoulli polynomials, etc.

UNIT-I

Arithmetical Functions and Dirichlet Multiplications: The arithmetical functions and their relations, The Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, Multiplicative functions, The Bell series of an arithmetical function and Dirichlet multiplications, Derivatives of arithmetical functions, The Selberg identity, The big oh notation, Euler's summation formula, Some elementary asymptotic formulas.

UNIT-II

Averages of arithmetical functions: The average order of divisor functions, The average order of Euler totient function, The average order of Mobious and Mangoldt functions, The partial sums of a Dirichlet product, Applications to the Mobius and Mangoldt functions. Some elementary theorems on distribution of prime numbers: Chebyshev's functions and their relations with $\pi(x)$ (The number of primes less than or equal to x), Some equivalent forms of the prime number theorem, Shapiro Tauberian theorem, The partial sums of the Mobius function.

UNIT-III

Brief sketch of an elementary proof of the prime number theorem, Functions periodic modulo k, Existence of finite Fourier series for periodic arithmetical functions, Ramanujan's sum and generalizations, The half-plane of absolute convergence of a Dirichlet series, Euler products, Analytic properties of Dirichlet series, Mean value formulas for Dirichlet series, An integral formulas for the coefficients and the partial sums of a Dirichlet series.

UNIT-IV

The Riemann zeta function and the L-function: Properties of the gamma function, Integral representation for the Hurwitz zeta function, Analytic continuation of the Hurwitz zeta function, Analytic continuation of the Riemann zeta function and the L-function, The functional equation for the Riemann zeta function, Properties of Bernoulli numbers and Bernoulli polynomials.

The course is covered by

1. Introduction to Analytic Number Theory, T. M. Apostol, Springer-Verlag, New York, 1976.

Books for reference

2. Analytic Number Theory: Exploring the Anatomy of Integers, Jean-Marie De Koninck, Florian Luca, American Mathematical Society, 2012

3. A Primer of Analytic Number Theory: From Pythagoras to Riemann, Jeffrey Stopple, Cambridge University Press, 2003.

ALGEBRAIC TOPOLOGY

Objective: The objective of this course is to augment to the course in topology using methods in Group theory and Ring theory. This course has a intrinsic application value having applications in network analysis. The homotopy theoretic method in differential equations and concept of homology and co-homology help in modelling in Physics. The students is exposed to Simplective Homology Groups, Co-homology and Homotopy in this course.

Expected Outcome: A students can take followup courses in co-homology theory, homotopy theory, homology theory, category theory. This course has application in Graph Theory and Networking also.

Unit-I

Motivation and Historical background,simplices,Orientation of simplex, simplicial Complexes and Simplicial maps. Review of Abelian Grourps, chains, Cycles, Boundary, Homology groups of simplicial complexes, examples of Homology groups,

The course is covered by Basic concepts of Algebraic Topology,categories and functors

Unit-II

, Homology groups of surfaces, zero dimensional homology, homology of a cone,relative homology , Simplicial Approximation, Barycentric Subdivision, induced Homomorphism, Exact homolgy sequences,zig zag lemma, Mayer Vietories sequences, Eilenberg Steenrod Axioms Axioms of Simplicial theory.,

Unit-III

Hom functor,Simplicial cohomology groups,Relative cohomology,cohomology theory,Cohomology of free chain complexes,Cup products,CW complexes the cohomology of CW complexes,Join of two complexes, Homology manifolds, Poincare duality, cap products

Unit-IV

Homotopic path, Homotopy equivalence,contractible space, deformation retract fundamental Groups, Covering Homotopy property for S' Examples of Fundamental groups, , Definition of covering spaces classification of covering spaces, Basic Properties of Covering Spaces.

The course is coverd by

1. Munkres, Elements of Algebraic Topology,Perseus Books; Re- vised ed. edition (11 December 1995)(1-9
2. Rotman, Algebraic Topology Springer Verlag, 1988.

   Reference Books for :

3. Croom, Basic Concepts of Algebraic Topology. Springer, 1978
4. Deo Algebraic Topology A primer Hindusthan Publishing house
5. Shatry A R Baisc Algebraic Topology

ADVANCED COMPLEX ANALYSIS

Objectives: *The course is aim to provide some exploration of Complex Analysis in different dimension. It is designed for the students who have completed a basic course in complex analysis in under graduate or post-graduate level. To register in the course the student must have reasonable mastery of analytic properties of functions.*

Expected Outcomes: *After completing this course, the students are expected to be able to*

- *Know the harmonic analogue of analytic functions, concept of sequence of analytic functions, normal families and idea of analytic continuation. This may help them to continue higher study in potential theory.*
- *Construct the entire as well as meromorphic functions and its properties. This will help to achieve a platform for deep research on growth of analytic functions and Nevannlina theory.*
- *Know the basic properties of elliptic functions due to Weierstrass. This will motivate to study analytic number theory and modular functions.*

Unit-I

Analytic continuation: direct analytic continuation, Natural boundary, function elements, complete analytic function, Harmonic functions in the disk, mean value theorem, Poisson integral, maximum/minimum principle for harmonic functions, Hrnack's inequalities, reflection principle for harmonic functions, Dirichlet problems for upper half plane, Sub-harmonic functions and their properties, Normal family.

Unit-II

Entire function: Product development, Zeros of entire functions, Weierstrass factorization theorem, exponent of convergence of zero of entire function, Genus, order and type of entire functions, Poincare theorem, Borel theorem, Hadamard theorem, entire functions with finitely many zeros.

Unit-III

Meromorphic functions: Mittag-Leffler representation of a meromorphic function, Gamma function, Digamma functions and their properties, Riemann zeta functions, Analytic continuation of zeta function, Poisson- Jensen's formula, Nevanlinna first fundamental theorem, order and type of meromorphic functions.

Unit-IV

Elliptic functions: doubly periodic functions, General properties of elliptic functions, Weierstrass P-function, σ-function and ζ-function, addition theorem for P and ζ-function, Elliptic Modular functions.

Course covered from:

1. Complex Analysis Selected Topics: Pure and applied mathematics. A series of Monographs and Textbooks, Dekker Publication, 1992, by Mario O. Gonzalez.
   (Ch3: 3.1 - 3.10; Ch4: 4.1 - 4.11; Ch5: 5.1 - 5.4, 5.27-5.34)
2. Complex Variable with Applications, Birkhauser, Springer India private limited, New Delhi, Indian Edition 2012 by S. Ponnusamy and H. Silverman
   (Ch10; Ch11:11.2; Ch13:13.1)

Reference books

1. Functions of one complex variable, J.B.Conway, Springer International Student Edition.
2. Complex Analysis, Lars V. Ahlfors, McGraw-Hill Int. Edition.
3. Lectures on theory of functions of complex variables, Vol. I, II, G. Sansone and J. Gerretsen, Noordhoff, Groningen, 1960
4. Function theory of one complex variables, Robert E. Greene and S.G. Krantz, 3rd Edition, AMS.

ADVANCED LINEAR ALGEBRA

Credits -4

OBJECTIVE:
This course is a second course in linear algebra.The objective of this course is to strengthen the linear algebraic tools of a student facilitating its application in various problems in computer science , engineering,economics and mathematics.
EXPECTED OUTCOME:
After taking this course the student will be able to
1.solve harder problems involving eigenvalues and eigenvectors.
2 solve problems of diagonalisation
3.understand the structure of normal operators
4. use QR alogorithms to find eigenvalues.

Prerequiste is a full course in linear algebra(no question in Exam) Thrust will be more on understanding,application and exercises

UNIT-I

.
Modules and The Structure of a Linear Operator

Modules,.Linear Independence.Submodules. Direct Sums. Spanning Sets. Homomorphisms. Free Modules..Quotient Modules. Quotient Rings and Maximal Ideals. Noetherian Modules. The Hilbert Basis Theorem. Free Modules over a Principal Ideal Domain. The Primary Decomposition Theorem. The Cyclic Decomposition Theorem
The Module Associated with a Linear Operator.Submodules and Invariant Subspaces. Orders and the Minimal Polynomial. Cyclic Submodules and Cyclic Subspaces. The Decomposition of V. The Rational Canonical Form

UNIT-II

.
Eigenvalues and Eigenvectors The Characteristic Polynomial of an Operator. Eigenvalues and Eigenvectors. The Cayley-Hamilton Theorem. The Jordan Canonical Form. Geometric and Algebraic Multiplicities. Diagonalizable Operators. Projections. The Algebra of Projections. Resolutions of the Identity. Projections and Diagonalizability. Projections and Invariance.

Matrix Inequalities

Positive Self-Adjoint Matrices , Monotone Matrix Functions ,Gram Matrices ,Schur's Theorem ,The Determinant of Positive Matrices ,Integral Formula for Determinants Eigenvalues, Separation of Eigenvalues Wie,landt-Hoffman Theorem 1A Smallest and Largest Eigenvalue , Matrices with Positive Se]f-Adjoint Part ,Polar Decomposition ,Singular Values ,Singular Value Decomposition

UNIT-III

QR Factorization ,Using the QR Factorization to Solve Systems of Equations , The QR Algorithm for Finding Eigenvalues , Householder Reflection for OR Factorization ,Tridiagonal Form ,Analogy of QR Algorithm and Toda Flow ,Moser’s Theorem ,

Inner product spaces :Norm and Distance. Isometrics. Orthogonality.Orthogonal and Orthonormal Sets. The Projection Theorem. The Gram-Schmidt Orthogonalization Process. The Riesz Representation Theorem, The Matrix of a Bilinear Form. Quadratic Forms. Linear Functionals. Orthogonal Direct Sums.

UNIT-IV

.

The Spectral Theorem for Normal Operators

The Adjoint of a Linear Operator. Orthogonal Diagonalizability.. Self-Adjoint Operators. Unitary Operators. NormalOperators. Orthogonal Diagonalization. Orthogonal Projections.,Orthogonal Resolutions of the Identity. The Spectral Theorem.Functional Calculus. Positive Operators. The Polar Decomposition of an Operator.

Metric Vector Spaces

Symmetric, Skew-symmetric and Alternate Forms. Quotient Spaces.Symplectic Geometry-Hyperbolic Planes. Orthogonal Geometry Orthogonal Bases. The Structure of an Orthogonal Geometry.Isometries. Symmetries. Witt's Cancellation Theorem. Witt's Extension Theorem. Maximum hyperbolic spaces.

The course is covered by

1. Steven Roman, Advanced Linear Algebra ,Springer GTM 1992
2. Peter D Lax ,Linear Algebra and its applications, WILEY second Edition 2007

COMBINATORICS

Objective: Combinatorial tools play a major role in any computational activity in mathematics starting from pure mathematics to computer science. They help in proving many results and identities in almost all branches of mathematics. This course aims at being a basic course introducing basic methods.

Expected Outcomes: A student who has completed this course can opt for new courses like combinatorial topology, combinatorial geometry and analysis in next semester or at higher level of doing mathematics.

Unit-I
Partial order sets, lattices, complements, Boolean algebra, Boolean expressions, counting principle, permutation, combination, multinomial theorem, set partitions, derangements, Stirling numbers.
Unit-II
Pigeon-hole principle, generalized inclusion-exclusion principle, Generating functions: Algebra of formal power series, generating function models, calculating generating functions, exponential generating functions, Recurrence relations, divide and conquer relations, solution of recurrence relations, solutions by generating functions.
Unit-III
Integer partitions, systems of distinct representatives, Polya theory of counting: Necklace problem and Burnside's lemma, cyclic index of a permutation group, Polya's theorems and their immediate applications.
Unit-IV
Latin squares, Hadamard matrices, Gaussian numbers and q-analogues, Mobius Inversion, combinatorial designs: t-designs, BIBDs, Symmetric designs.

Book for Reference:

1. Lint, J. H. van, and Wilson, R. M.: "A Course in Combinatorics", Cambridge University Press , (2nd Ed.) , 2001.
2. V. K. Balakrishnam, Theory and problems of combinatories, McGraw-Hill, 1994.
3. Sane, S. S.: "Combinatorial Techniques", Hindustan Book Agency , 2013
4. Brualdi, R. A.: "Introductory Combinatorics", Pearson Education Inc. (5th Ed.),2009
5. Krishnamurthy, V.: "Combinatorics: Theory and Applications", Affiliated East-West Press, 1985.
6. Hall, M. Jr.: "Combinatorial Theory", John Wiley & Sons (2nd Ed.), 1986.

Cryptography

Objectives:

- Enable the students to learn fundamental concepts of cryptography and utilize these techniques in computing systems.
- To acquire knowledge on standard algorithms used to provide confidentiality, integrity and authenticity.
- To understand the various key distribution and management schemes.
- To understand how to deploy encryption techniques to secure data.

Expected Outcomes: Upon successful completion of this course students will be able to

- Have a strong background of cryptography which has diverse practical applications.
- Encrypt and decrypt messages using block ciphers, sign and verify messages using well known signature generation and verification algorithms.
- Analyse existing authentication and key agreements.
- Develop their skills in the programming of symmetric and/or asymmetric ciphers and their use in the networks.

Unit-I

Some Simple Cryptosystems: The Shift Cipher, Substitution Cipher, Affine Cipher, Vigenere Cipher, Hill Cipher, Permutation Cipher, Stream Cipher. Cryptanalysis of Affine Cipher, Cryptanalysis of Substitution Cipher, Cryptanalysis of Vigenere Cipher, Cryptanalysis of Hill Cipher.

Unit-II

Linear Cryptanalysis, Differential Cryptanalysis, Basic Encryption and Decryption, Encryption Techniques, Characteristics of Good Encryption Systems, International Data Encryption Algorithm, Shannon's Theory: Elementary probability theory, perfect secrecy, Entropy, Properties of Entropy, Product cryptosystems.

Unit-III

Public Key Cryptography, The RSA Cryptosystem, Primality Testing, Square roots modulo m, Factoring Algorithms, Other attacks on RSA, Finite fields, Elliptic Curves.

Unit-IV

Secret Key Cryptography, Diffie-Hellman Key Pre-distribution, Key Distribution Patterns, Diffie-Hellman Key Agreement. Pseudo-random Number Generation, BBS generator, Probabilistic Encryption, Digital Signatures , One-time Signatures, Rabin and ElGamal Signatures Schemes, Digital Signature Standard (DSS).

The course is covered by

1. Stinson, D. R., CRYPTOGRAPHY: Theory and Practice, CRC Press, 1995.
2. Stallings, W., Cryptography and Network Security, 5 th Edition, Pearson, 2010.

Book for reference

3. Koblitz , N., A Course in Number Theory and Cryptography, Graduate Texts in Mathematics, New-York: Springer-Verlag, 1987.

DATA STRUCTURE

Objective: Data Analysis has become the spine of all computational and statistical research having major contributions in social sciences, computer science and day to day life. Data structure is taught as a first course in Data analysis to expose the students to basics.

Expected outcomes: The students completing this course in Data structure can go for advance courses in Data structure, Data base management and can be introduced to machine learning and artificial intelligence courses if he has already taken some programming courses.

Unit-I

What are data structures, Java Refesher, JAVA refresher and generics, Analysis Tools and Techniques, Algorithm Analysis, Mathematical Background, Model, Running Time Calcula- tions

Unit-II

Abstract Data Types (ADTs), vector and list in the STL, Linked lists and Iterators, Stacks and Queues, The Stack ADT, The Queue ADT

Unit-III

Binary Trees, The Search Tree ADTBinary Search Trees, AVL Trees, Splay Trees, B-Trees, Hash Function, Separate Chaining, Hash Tables Without Linked Lists, Rehashing

Unit-IV

Priority Ques, Models, Simple Implementations, Binary Heap, Applications of Priority Queues, d-Heaps, Sorting, Graph

Books for Reference:

1. M. A. Weiss. Data Structures and Algorithm Analysis in C++ (3rd edition), by Addison- Wesley
2. Alfred V. Aho, Jeffrey D. Ullman, John E. Hopcroft. Data Structures and Algorithms. Addison Wesley, 1983.
3. Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein. Introduction to Algorithms. McGraw-Hill, 2001.
4. Donald E. Knuth. The Art of Computer Programming, Volumes 1-3. Addison-Wesley Professional, 1998.

DISCRETE DYNAMICAL SYSTEMS

Objective: The evolution of a point in time under a map has caused major studies in Mathematics introducing Discrete Dynamical System. The concepts of Fatou Sets, Julia Sets, Cellular automata, Fractals, Horseshoes, Hyperbolic dynamics have added many new results and has evolved many new modelling techniques. The objective of the course is to expose the students to this new direction.

Expected Outcome: This first course will train the students in understanding topological dynamics essentially and will help in opting for new course on Fractal Geometry, Bifurcation, Cellular Automata, Julia Set etc. later. This will also help in going for a second course in continuous dynamical system and its application to differential equations.

Unit-I

Phase Portraits, Periodic Points and Stable Sets, Sarkovskii's Theorem, Hyperbolic, Attracting and Repelling Periodic Points. Families of Dynamical Systems, Bifurcation, Topological Conjugacy. The Logistic Function, Cantor Sets and Chaos, Period-Doubling Cascade

Unit-II

More examples, Rotations, Horse shoes ,solenoid Limit sets and recurrence, topological transi- tivity,topological mixing, expansiveness

Unit-III

Topological entropy, examples . Symbolic Dynamics. Sub shifts and codes, sub shifts of finite type, topological entropy of an SFT, Newton's Method. Numerical Solutions of Differential Equations

Unit-IV

Complex Dynamics, Quadratic Family, Julia Sets, Mandelbrot Set. Topological Entropy, At- tractors and Fractals, Theory of Chaotic Dynamical System.

The course is covered by

1. Richard M. Holmgren: A First Course in Discrete Dynamical Systems,Springer Verlag,1996.
2. Walter An introduction to ergodic theory Springer
3. Devaney : Introduction to Chaotic Dynamical Systems.

FLUID MECHANICS

Objectives:
The objective of this paper to make this student familiar with concept of Fluid Mechanics and the property of ideal fluid flow past a rigid body

. Expected outcomes: After completion of this course the students will be able to

1. Recognize the property of fluids with examples.
2. Identify different equations of fluid motion.
3. Demonstrate a working knowledge of fluid flow.

Unit-I :

Basic concepts: Scope of fluid Mechanics, Definition of fluid, Continuum hypothesis, stress in a fluid at rest and in motion,. Relation between stress rate and strain components. Fluid properties, Fluid statics.

Unit-II:

Method of describing fluid motion, velocity and acceleration of fluid particles, Equation of continuity, energy and momentum. Linear momentum equation, Euler's equation of motion along a stream line.

Unit-III:

Navier Stoke's equation, Energy equation, Reynold's experiments, Development of laminar and turbulent flows in pipes. Approximate solution for incompressible flow, Poiseullie's flow, vorticity and circulation in viscous flow. Unit-IV: Boundary layer Theory, Definition of boundary layer equation, Displacement, momentum and energy thickness, separation of boundary layer. 39

The course is covered by:

1. M.D. Raisinghania, Fluid Dynamics, S Chand Publication.
2. J.L. Bansal, Fluid Mechanics, IBH Publications.

Books for References:

1. John F. Douglas, Gariorck, Swaffield, B Jack , Fluid Mechanics. 5th Edition, Pearson.
2. F. M. White, Fluid Mechanics, Tata McGraw Hill, (2008)

FOURIER ANALYSIS

Objective: This is a basic course in Fourier series. It is designed so as to know the conditions under which Fourier series expansion of a function exists, to study on criteria for convergence and summability of Fourier series. It gives an idea how knowledge of Fourier series helps to define Fourier transform and to study its convergence and summability. Basic of Discrete Fourier series as also introduced. The pre requisite courses for this is real analysis, complex analysis and Functional analysis. This course carries its importance since more than century because of its vast application and analytic beauty.

Expected Outcomes: This course prepare a students to go for courses in Fourier Trnasform, Wavelets, Image Processing and Harmonic Analysis. Using Dirichlet conditions students can evaluate infinite series. Students can directly be exposed to state of the art research problem in this area.

UNIT-I

Fourier Series: Trigonometric series, Fourier Series, Fourier Sine Series and Cosine

Series, Other type of whole range series, Half range series, Uniqueness of Fourier series, The Riemann- Lebesgue Lemma, Dirichlet Kernel, Criterion and tests for Pointwise convergence of Fourier Series, Dirichlet's Pointwise convergence theorem.

UNIT-II

Convergence of Fourier Series:, Uniform convergence, The Gibb's phenomenon, Termwise inte- gration, Termwise differentiation, Cesaro summability, Toeplitz summability, Abel summabil- ity, Fejer Theory, Smoothing effect of (C,1) Summability, Lebesgue's Pointwise convergence.

UNIT-III

Fourier transform: Finite Fourier transform, Convolution on T, Fourier transform, Basic properties of Fourier transform, The Fourier map, Convolution on R, Inversion, Criterion and tests for convergence of Fourier integral formula, (C,1) summability for Integrals.

UNIT-IV

The Fejer-Lebesgue Inversion theorem, Integrability of Fourier Transform, Transforms of Deriva- tive and Integrals, Fourier Sine and cosine transforms, Parseval's identities, The Plancheral theorem, Discrete Fourier transform(DFT), Inversion theorem for the DFT, Parseval's Identi- ties.

Book for Reference:

1. G. Bachman, L. Narici, E. Beckenstein, Fourier and Wavelet Analysis, Springer,2000.
2. I.N. Sneddon, Fourier Series by Dover Publications, 1969.
3. Lokenath Debnath and Dambaru Bhatta, Integral Transforms and Their Applica- tions, Chapman and Hall/CRC, New York, 2007.
4. Fourier Analysis, T.W. Korner, Cambridge University press, 1988.
5. Elian M Stein and Rami Shakarchi, Fourier Analysis: An Introduction, Princeton Lectures in Analysis, Princeton University Press, Princeton and Oxford, 2003.

GEOMETRIC FUNCTION THEORY

Unit-I

Theory of univalent functions: basic properties, area theorems, growth theorem, distortion theorem, rotation theorem, Koebe's 1/4 theorem, coefficient problems.

Unit-II

Subclasses of the function class S: Convex function, Starlike function, Close-to-Convex function, Spirllike function, Class with positive real parts, Typically real functions and their generalizations.

Unit-III

Subordination: basic principle, coefficient inequalities, memorization, univalent subordinate functions, Integral mean, Bernstein theorem.

Unit-IV

H^p space: Harmonic functions, boundary behavior of Poisson-Stieltjes integral, Boundary value of H^p functions, zeros of analytic functions, Subharmonic functions, Hardy's convexity theorem, Blaschke product, Mean convergence to boundary value of H^p functions, Canonical factorization theorem.

Course covered from:

1. Univalent functions, Springer-Verlag, New York, by P.L.Duren,
2. Univalent functions, Vol.-I, II, by A. W. Goodman
3. Theory of H^p Space, Academic Press, New York, 1970, by P.L.Duren

Reference books

4. Geometric Function Theory in One and Higher Dimensions, Marcel Dekker, New York, 2003, by I.Graham, G. Kohr.
5. Linear Problems and Convexity Techniques in Geometric Function Theory, Pitman Adv. Publ. Program, Boston-London-Melbourne, 1984, by D. J. Hallenbeck, T. H. MacGregor.
6. Topics in Hardy Classes and Univalent Functions, Birkhauser Verlag, 1994, by M. Rosenblum, J. Rovnyak.

GRAPH THEORY

Objectives: Graphs are used to model networking problems in physical and biological sciences etc. As an essential tools in computer and information sciences, the concepts in Graph Theory address problems of social media, linguistics, chemical bonds, computational neuro science, market and financial analysis, communication system, data organisation, flows and links. The objective of this course is to introduce the basic of Graph Theory to students.

Expected Outcomes: A course in Graph Theory is prerequisite to almost all courses and research in computer science. Besides it has applications to other branches in mathematical sciences. A student can opt for Matroid theory, Network Analysis, Algorithm and Data Analysis courses after completing this course.

Unit- I

Definition and Examples, Connectedness, Walk, Path circuits, Euleriar graph, Hamiltonian graph, Some application.

Unit- II

Trees: Elementary proportion of trees, Enumeration of trees, More application. Cut sets:- Fundamental circuits and cut-sets, network flows, 1-isomorphism, 2-isomorphism.

Unit- III

Planarity:- Kuratowski two graphs, detection of planarity, geometric dual, thickness and cross- ing.

Unit- IV

Coloring problems, chromatic number, four color problem. Directed graph: Digraphs and bi- nary relations, Euler digraphs.

Books for Reference:

1. N. Deo Graph Theory and its Application to Engineering and Computer Science, PHI, 1979.
2. F. Harary Graph Theory, Addison Wesley Publishing company, 1969.
3. R. J. Wilson Introduction to Graph Theory, Longman Group Ltd., 1985 .

MECHANICS

Objective: This course is aim to be a second course to the existing undergraduate courses. They will introduce Lagrangian and Hamiltonian mechanics with all necessary Geometric pre requisites like differential Geometry and Symplectic Geometry.

Expected Outcome: This course can be followed by courses in Integrable models, foundation of Mechanics, Celestial Mechanics etc. This prepares an adequate mathematical background for understanding any research papers in Mechanics.

Unit- I

Newtonian Mechanics : Experimental facts, Investigation of Equation of Motion.

Unit- II

Lagrangian Mechanics : Variational Principles, Lagrangian Mechanics on Manifolds,

Unit- III

Oscillations, Rigid Bodies.

Unit- IV

Himiltonian Mechanics : Differential forms, SimplCetic structure on manifolds

Books for Reference

1. Ordinary Differential Equations V. I. Arnold, Springer, 1992.
2. Mathematical Methods in Classical Mechanics" by V.I. Arnold. Springer Verlag, 1978.

MATHEMATICAL MODELING

Objective: Apart from getting exposed to pure forms of mathematical abstractions, the objective of the course is to expose the students to hands on state of the art methods in modeling, real life situations in industry, Biology and nature.

Expected Outcomes: This course is based on modelling using elementary mathematics and differential equations. This can lead to more modelling courses using stochastic process, Discrete dynamical system, Optimization methods, finite elements, wavelets learning techniques etc.

UNIT-I

Need, Techniques, Classification and Characteristics of Mathematical Modeling. Mathematical modeling through first order ODE: Linear and nonlinear growth and decay model, Compartment model, Model of geometrical problems, Prey-Predetor model through delay-differential equations.

UNIT-II

Mathematical modeling through system of first order ODE: Modeling on population dynamics, Epidemic model, Compartment models, Modeling on Economics, Model in medicine, Models in arms race and battles.

UNIT-III

Mathematical modeling through second order ODE: Modeling of Planetory motion, circular motion, Motion of satellite. Modeling on rectilinear motion, freely falling body, oscillation of pendulum. Mathematical modeling on the Catenary.

UNIT-IV

Mathematical modeling through integral equations. Mathematical modeling through PDE: One dimensional wave equation, One dimensional heat equations, Two dimensional heat equations and Laplace equations.

Books for Reference

1. D. N. Burghes- Modeling through Differential Equation, Ellis Horwood and John Wiley.
2. C. Dyson and E. Levery, Principle of Mathematical Modeling, Academic Press New York.
3. Giordano, Weir, Fox, A First Course in Mathematical Modeling 2nd Edition, Brooks/ Cole Publishing Company, 1997.
4. J. N. Kapur, Mathematical Modeling, Wiley Eastern Ltd. 1994.
5. B. Barnes, G. R. Fulford, *Mathematical Modeling with Case Studies, A Differential Equation Approach using Maple and Matlab,* 2nd Ed., Taylor and Francis group, London and New York, 2009.

Number theory and Foundation of Cryptography

This course is an introduction to number theory and its applications to modern cryptography. Number theory, one of the oldest branches of mathematics, is about the endlessly fascinating properties of integers. The students will also learn how number theory is used in public key cryptography to securely transmit information over the internet. This leads naturally to discussions of factoring, primality testing, and the discrete logarithm problem.

UNIT-I

Divisibility: The Division Algorithm, Prime and Composite Numbers, Fibonacci and Lucas Numbers, Fermat Numbers, Greatest Common Divisors, The Euclidean Algorithm, The Fundamental Theorem of Arithmetic, Least Common Multiple, Linear Diophantine Equations, Congruences, Linear Congruences, The Pollard Rho Factoring Method.

UNIT-II

Systems of Linear Congruences: The Chinese Remainder Theorem, General Linear System, 2x2 Linear systems, Wilson's Theorem, Fermat's Little Theorem, Euler's Theorem, Perfect numbers, Mersenne Primes, Primitive Roots and Indices: The order of a positive integers, Primality Tests, Primitive Roots for Primes, The algebra of Indices.

UNIT-III

Quadratic Congruences: Quadratic Residues, The Legendre Symbol, Quadratic Reciprocity, The Jacobi Symbol, Finite Continued Fraction, Pythagorean Triangles, Fermat's Last Theorem, Pell's Equation.

UNIT-IV

Cryptography: Affine Ciphers, Hill Ciphers, Exponentiation Ciphers, Public key cryptography, The RSA Cryptosystem, Diffie-Hellman Key Exchange Algorithm, Knapsack Ciphers.

Books Recommended

1. Elementary Number Theory with Applications, Thomas Koshy, 2^{nd} edition, Academic Press, An Imprint of Elsevier, USA, 2007.
2. A Course in Number Theory and Cryptography: Neal Koblitz, SpringerVerlag, New York Inc. May 2001.
3. Cryptography. Theory and Practice, D.R. Stinson, CRC Press, Boca Raton, 2002.

OPERATOR THEORY

Objective: The dominance of operator methods in foundation of quantum mechanics and the success of spectral analysis and scattering methods and the evolution of operator algebras has found operator theory as an essential course at post graduate level. The objective of the course is to introduce basic operator theoretic methods as a second course to functional analysis.

Expected Outcomes: This course prepare a student to take a second course in operator algebra, Differential operators, Spectral theory, scattering theory, Fundamental solutions quantum probability etc. This is highly applicational. This course open ways to different research areas in this branch particularly and also in the area of functional analysis broadly, like representation theory , operators on different function spaces etc..

Unit- I

Banach Algebra : Introduction , Complex homomorphism Basic properties of spectra, Com- mutative Banach Algebra : Ideals, Gelfand transform, Involution, Bounded operator.

Unit-II

Bounded Operator : Invertibility of bounded operator, Adjoints, Spectrum of bounded operator, Fundamentals of spectral Theory, Self adjoint operators, Normal, Unitary operators, Projection Operator, introduction to complex measure, Resolution of the Identity.

Unit- III

Spectral Theorem, Eigen Values of Normal Operators, Positive operators, Square root of Positive operators, Partial Isometry, Invariant of Spaces, Compact and Fredholm Operators, Integral Operators.

Unit- IV

Unbounded Operators : Introduction, Closed Operators, Graphs and Symmetric Operators, Cayley transform, Deficiency Indices, Resolution of Identity, Spectral Theorem of normal Operators, Semi group of Operators.

Books for Reference

1. Walter Rudin, Functional Analysis, Tata McGraw Hill, 2010.
2. Weidman J, Linear Operators on Hilbert Spaces, Springer, 1980
3. R.G. Douglas, Banach Algebra Techniques in Operator Theory, Springer,1997.
4. Gohberg and Goldberg Basic Operator Theory, 2001.
5. M. Schecter, Principle of Functional Analysis, American Mathematical society, 2002.
6. Akhietzer and Glazeman: Theory of Linear Operator, Vol I, II ,Pitman Publishing House,1981.

THEORY OF COMPUTATIONS

Objective

To have an understanding of finite state and pushdown automata, regular languages and context free languages, to know relation between regular language, context free language and corresponding recognizers; to gain knowledge about Pushdown Automata, Turing machines, and Intractability

UNIT – I

Introduction to formal proof – Additional forms of proof – Inductive proofs – Finite Automata (FA) – Deterministic Finite Automata (DFA) – Non-deterministic Finite Automata (NFA) – Finite Automata with Epsilon transitions.

UNIT – II

REGULAR EXPRESSIONS AND LANGUAGES (10 Hours) Regular Expression – FA and Regular Expressions – Proving languages not to be regular – Closure properties of regular languages – Equivalence and minimization of Automata.

UNIT – III:

CONTEXT-FREE GRAMMAR, AND PROPERTIES (16 Hours) Context-Free Grammar (CFG) – Parse Trees – Ambiguity in grammars and languages – Definition of the Pushdown automata – Languages of a Pushdown Automata – Equivalence of Pushdown automata and CFG, Deterministic Pushdown Automata. Normal forms for CFG – Pumping Lemma for CFL - Closure Properties of CFL – Turing Machines – Programming Techniques for TM.

UNIT – IV:

UNDECIDABILITY A language that is not Recursively Enumerable (RE) – An undecidable problem that is RE – Undecidable problems about Turing Machine – Post's Correspondence Problem - The classes P and NP.

The course is covered by:

1. John E. Hopcroft, Rajeev Motwani and Jeffery D. Ullman, Automata Theory, Languages, and Computation (3rd. Edition), Pearson Education, 2008. 46

Books of Reference:

1. H.R.Lewis and C.H.Papadimitriou, Elements of The theory of Computation, Second Edition, Pearson Education/PHI, 2003
2. Michael Sipser, Introduction to the Theory of Computation, Books/Cole Thomson Learning, 2001.
3. J.E. Hopcroft and JD Ullman, Introduction to Automata Theory, Languages, and Computation, Addison-Wesley, 1979.

WAVELETS

Unit- I

Review of Fourier Analysis, Elementary ideas of signal processing, From Fourier Analysis to wavelet Analysis, Windowed Fourier Transforms : Time frequency localization, The reconstruction formulae.

Unit- II

Multiresolution analysis, Construction of Wavelets from MRA, construction of compactly supported wavelets, Band limited Wavelets, Franklin wavelets on real line, Introduction to spline analysis, spline wavelets on real line,

Unit- III

Discrete transforms and algorithms, Discrete Fourier transform and the fast Fourier transform, Discrete cosine transform and the fast cosine transform, Decomposition and reconstruction algorithm for Wavelets, Application of wavelets

Unit - II

Representation of Functions by Wavelets, Characterizations of function spaces using wavelets, Non existence of smooth wavelets in H^2(R),Theory of Frames, The reconstruction formula, Balian-Low theorem for frame ,frame from translation and dilation , Smooth frames for H^2(R), Orthogonal Wavelets.

The course is covered by

1. Harnandez, E. A first Course in waveletes, CRC
2. Chui An Intruduction to Wavetets, Academic Press.

Books for reference : -

3. Daubechies Ingrid Ten Lectures on Wavetets.
4. Kaiser, G. A friendly guide to Wavelets, Bikhauser 1994.
5. Kahane & Lemaire Rieusset Fourier Series & Wavelets Gordon & Breach.

FRACTAL GEOMETRY

Objectives: The objective of this course is to provide a comprehensive understanding of the fundamental concepts and mathematical principles underlying fractal geometry. Students will explore the theoretical framework of fractals, including self-similarity, scaling, and dimension, and how these concepts apply to various fields such as physics, biology, and computer science.

Expected outcome: After studying this course, the student will be able to

CO1: get the idea of an application of Measure Theory in this course.

CO2: gain a solid understanding of fundamental concepts, properties, and measurement techniques related to fractals.

CO3: be proficient in constructing iterated function systems (IFS) and analysing advanced fractals such as the Mandelbrot and Julia sets.

CO4: be able to apply fractal geometry principles across various fields, enhancing their analytical skills and problem-solving capabilities in real-world scenarios.

Unit 1

Introduction to Fractal Geometry: Definition and key properties of fractals, Examples of fractals, Historical development and key contributors (e.g., Benoît B. Mandelbrot), Applications and relevance in various fields, Self-similarity, Fractal dimension.

Unit 2

Fractal Dimension and Measurement: Box-counting dimension, Hausdorff dimension, Comparison and calculation methods, use of scaling laws and algorithms to determine fractal dimension, different fractal dimensions.

Unit 3

Iterated Function Systems (IFS) and Applications: Definition and construction of IFS, Attractors and fixed points, Stability and chaos in IFS, Relationship between IFS and other fractal constructions, Cantor sets, Cantor dusts, Koch Snowflake, Sierpinski triangle.

Unit 4

Graphs of functions: Dimensions of graphs, the Weierstrass function and self-affine graphs, Iteration of complex functions: Construction of Julia set and Mandelbrot set.

Course is covered by:

1. Falconer, Kenneth. *Fractal geometry: mathematical foundations and applications*.3rd edn, John Wiley & Sons, 2014.
   (Introduction, Chapter 2: 2.1, 2.2, Chapter3: 3.1, 3.2, 3.3, Chapter4: 4.1, Chapter 9: 9.1,9.2 ,9.3, Chapter 11: 11.1, Chapter 14: 14.1, 14.2, 14.3)

Books for references:

2. Barnsley, Michael F. *Fractals everywhere*. Academic press, 2014.
3. Falconer, Kenneth. *Fractals: A very short introduction*. OUP Oxford, 2013.
4. Edgar, Gerald A., and Gerald A. Edgar. *Measure, topology, and fractal geometry*. Vol. 2. New York: Springer, 2008.

Numerical Solution of Partial Differential Equations

Objective: The objective of this course is to familiarize the student to finite element method. FEM is one of the most powerful and versatile methods available for solving PDEs encountered in engineering and physics. FEM in the time since has developed into a commonly used tool in such diverse areas as fluid mechanics, electrostatics, magnetohydrodynamics, neutron diffusion, and heat transfer. The popularity of the FEM has been in part due to the remarkable success most users have had with the method, although in many cases where complicated, irregular domains must be treated, the FEM is the only recourse.

After completing the course the student will be able to

CO1: understand the theory of distribution, Sobolev space and weak solution of boundary valued

Elliptic problem.

CO2: get the idea to implement of FEM for one dimensional model problem and derive the error bound for approximation solution.

CO3: understand the theories and implication of FEM for second order Elliptic problem.

CO4: acquire knowledge on the theories and implication of FEM for second order parabolic problem.

Unit I

Distribution, derivative in the sense of distribution, Convolution on distribution, Schwarz Space, Sobolev Space, weak derivative, weak solution of elliptic boundary value problem.

Unit II

Introduction to FEM for Elliptic Problems: Variational Formulation of one-dimensional model problem, FEM for the model problem with piecewise linear functions. Error estimate for FEM for the model problem, FEM for passion equations, geometric interpretation of FEM, Neumann problem.

Unit III

Abstract formulation of FEM for elliptic problem: The continuous problem, an error estimate, the energy norm, regularity results , some examples of finite elements, Interpolation with piecewise linear functions in two dimensions, interpolation with polynomials of higher degree, error estimate for elliptic problems in two dimension. On the regularity of exact solution.

Unit IV

FEM for Parabolic problem, One dimensional model problem, Semi Discretization in space, Discretization in space and time, The backward Euler and Crack- Nikolson Methods, Error Estimates for fully discrete approximation.

Text Books

1) S. Keshavan, Topics in Functional Analysis and Applications, second edition, NewAge publication. (Unit-I)

2) C. Johnson, Numerical solutions of partial differential equations by the finite element method, Cambridge University Press. (Unit II, III, IV)

BOOKS FOR REFERENCE

3) S.C Brenner, L. R. Scott, The mathematical Theory of Finite Element Methods, second Edition, Springer

4) P.G. Ciarlet, The finite element method for elliptic problems, SIAM